Dominance move: concept, advances and applications in multi- and many-objective optimization problems

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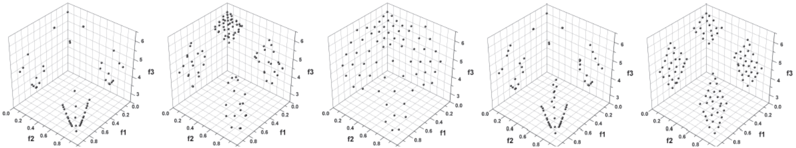
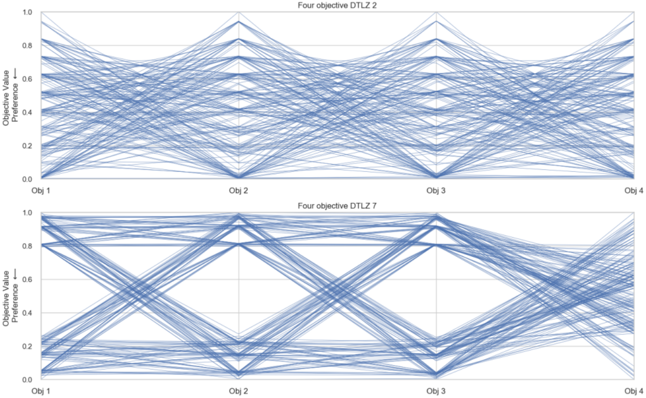
Outline

- Multi-objective and many-objective problems are common in real cases;
- How to summarize approximation sets, taking into account:
  - Location;
  - Shape;
  - Distribution.



Introduction

- **Graphical techniques:** multi-objective problems;



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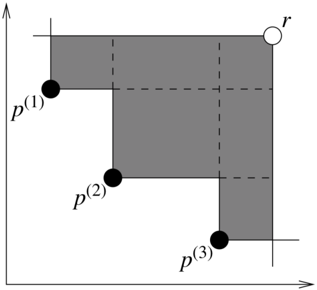
Introduction

- **Quality indicators:** multi-objective and many-objective problems;
- Some well know quality indicators: Hypervolume, IGD and ϵ-indicators;
- Most quality indicators have drawbacks: reference point, reference sets, sensitive to extreme point, lack of information, computational cost, and others;



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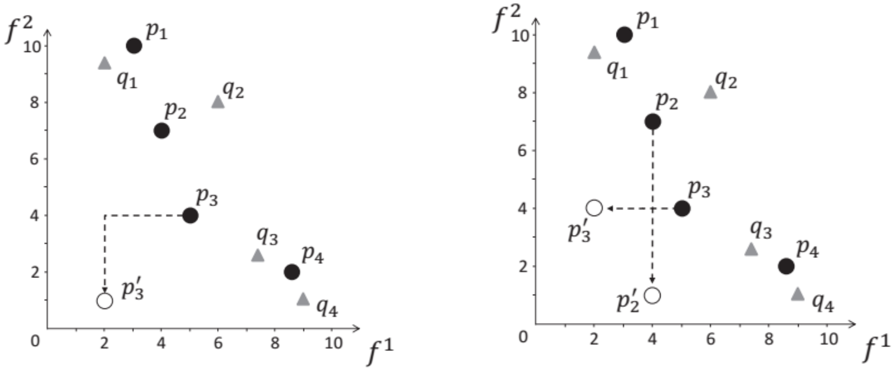
Dominance Move - DoM

- Dominance Move (DoM) is a new binary quality indicator:
  - It does not require any pre-defined set of points, such as, a reference point or a reference set;
  - It is not affected by dominance resistant solutions, unlike that in HV;
  - There is no lack of information, such as in ϵ-indicators.
- DoM calculates the minimum total move of members of one set so that all elements in another set are to be dominated or identical to at least one member of the first set.

Dominance Move - DoM

Example DoM(P,Q):



Dominance Move - DoM

Definition

Dominance Move (DoM): Consider that $\boldsymbol{P}$ and $\boldsymbol{Q}$ are two sets of points, with $\boldsymbol{p}_i$ points $i \in \{1, \ldots, |\boldsymbol{P}|\}$ and $\boldsymbol{q}_j$ points $j \in \{1, \ldots, |\boldsymbol{Q}|\}$. The dominance move of $\boldsymbol{P}$ to $\boldsymbol{Q}$, $DoM(\boldsymbol{P}, \boldsymbol{Q})$, is the minimum total distance of moving points of $\boldsymbol{P}$, such that the moved set $\boldsymbol{P'} = \{\boldsymbol{p}'_1, \boldsymbol{p}'_2, \ldots, \boldsymbol{p}'_{|\boldsymbol{P}|}\}$ (with some or all $\boldsymbol{p}'_i$ are allowed to be infeasible) from $\boldsymbol{P} = \{\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_{|\boldsymbol{P}|}\}$ dominates $\boldsymbol{Q}$ and that the total move from $\boldsymbol{P}$ to $\boldsymbol{P'}$ must be minimum [1].

The problem is to find $\boldsymbol{P'}$ from $\boldsymbol{P}$ and the total move from $\boldsymbol{P}$ to $\boldsymbol{P'}$, denoted as $d(\boldsymbol{p_i}, \boldsymbol{p_i'})$, must be minimum:

$$DoM(\boldsymbol{P}, \boldsymbol{Q}) = \underset{\boldsymbol{P'} \prec \boldsymbol{Q}}{minimize} \sum_{i=1}^{|\boldsymbol{P}|} d(\boldsymbol{p_i}, \boldsymbol{p_i'})$$

Dominance Move - DoM

- There is a question regarding its calculation computational cost;
- Algorithm for dominance move calculation was proposed [2]. It is applied only in bi-objectives problem sets;
- DoM authors proposed a calculation approach, but without correct calculation guarantees [3].

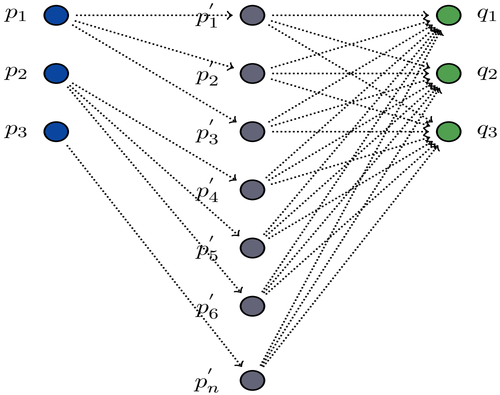
DoM as an assignment problem

DoM formulation as an assignment problem, and its solution using a mixed-integer programming method [4]:

- An assignment formulation approach;
- A comparative discussion between ϵ-indicators and DoM;
- Some experiments using classical problems sets and some evolutionary algorithms (IBEA, NSGA-II, and SPEA2) using DoM and others quality indicators, such as HV and IGD.

Dominance move indicator and its assignment formulation

- A graph that represents the assignment approach:



Dominance move indicator and its assignment formulation

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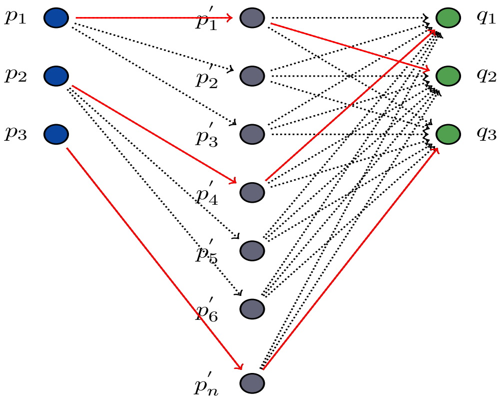

Dominance move indicator and its assignment formulation

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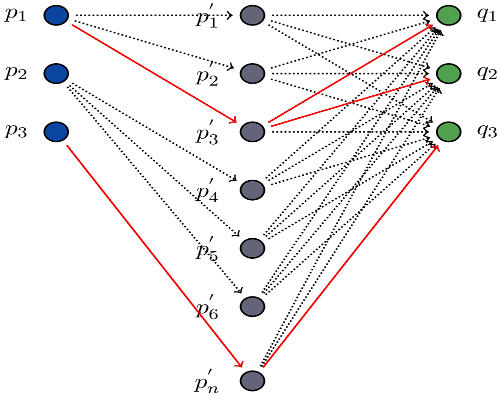
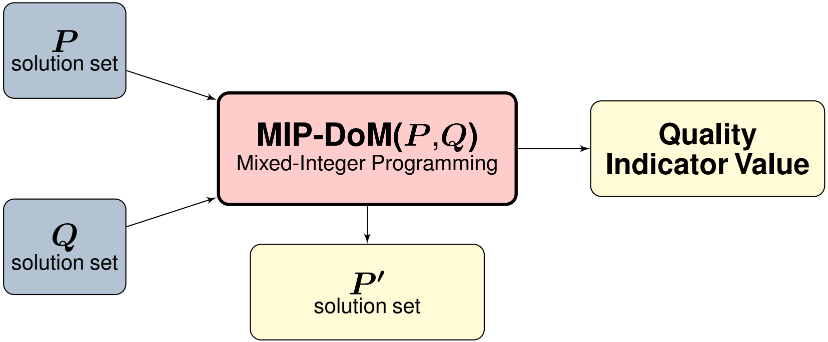


Dominance move indicator and its assignment formulation

The assignments necessary to calculate the formulation turns it prohibitive when the number of solutions in the non-dominated set increase.

MIP-DoM preamble

MIP-DoM approach :



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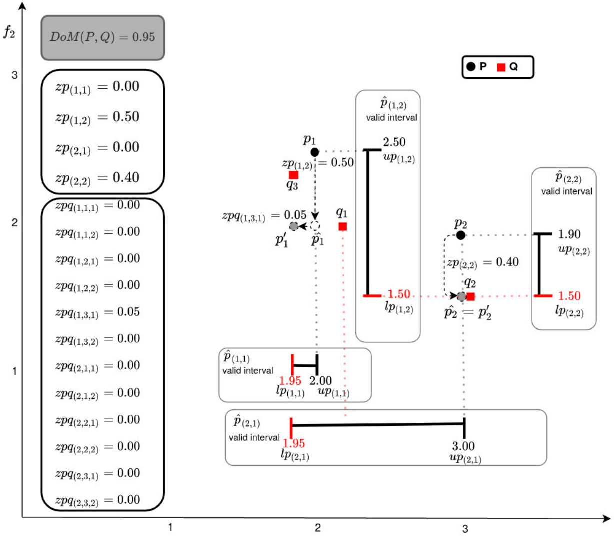
MIP-DoM Intuition - Example

- Introduction
- Introduction
- Introduction
- Related works
- **DoM assignment formulation and MIP-DoM**
- Experiments
- Conclusion
- Future research and final comments
- References



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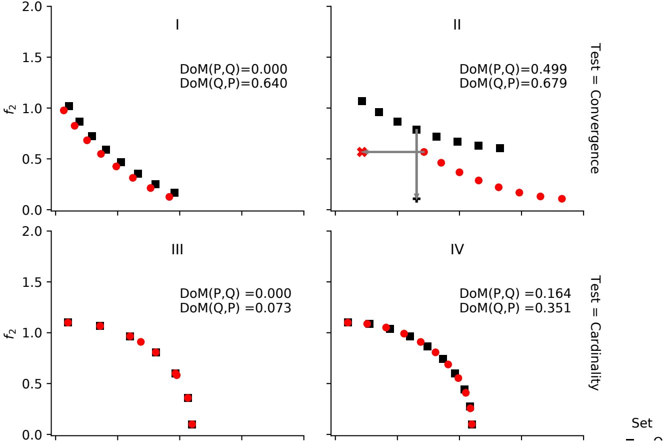
Experiments design

- Assessing some solution sets features (*convergence*, *spread*, *uniformity*, and *cardinality*);
- Comparing DoM and ϵ-indicators;
- Testing on more complex test problems - 3 to 30 objectives using till 400 solutions in each set;
- Parametric studies;
- Time-complexity estimates;
- DoM as a running quality indicator.

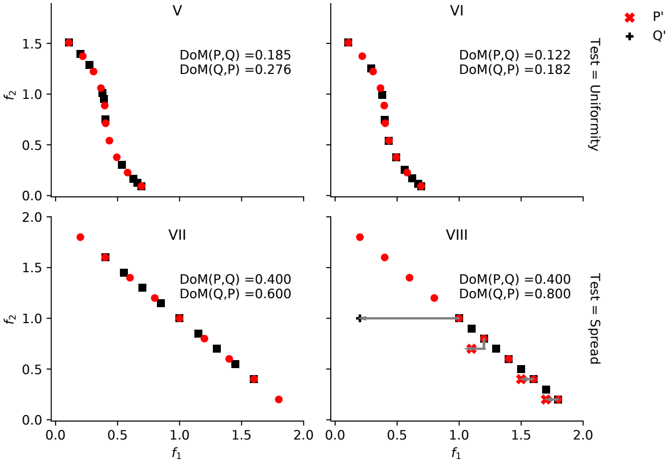
Solution sets features

Bi-objective cases



Solution sets features

Bi-objective cases

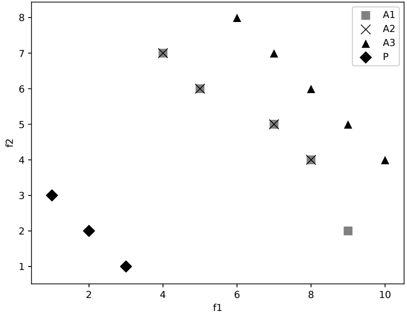


DoM and ϵ-indicators

They have a similar interpretation.

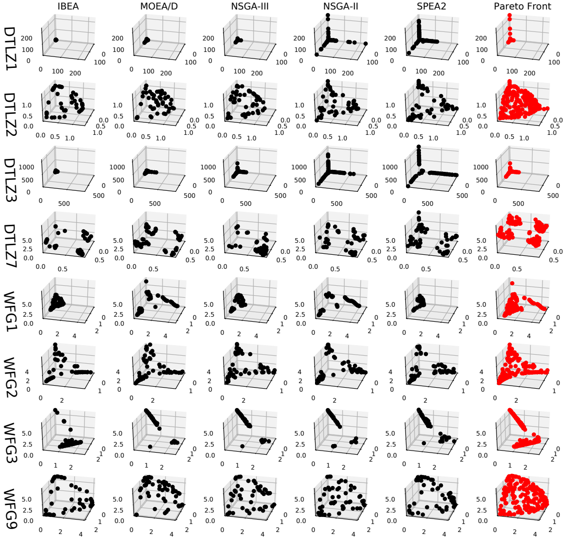
- DoM presents greater values than ϵ-indicators;
- There is an information loss: ϵ-indicators are only related to one particular solution and only one objective in whole solution set,e.g.: two 10-objective solutions, $p_1 = \{0, 0, 0, .., 1\}$ and $q_1 = \{1, 1, 1, .., 0\}$. The ϵ-additive(p_1, q_1) = ϵ-additive(q_1, p_1) = 1.

Quality indicator	P solution sets	Q solution sets			
		A_1	A_2	A_3	P
ϵ-additive	A_1	0.000	0.000	-1.000	4.000
	A_2	2.000	0.000	0.000	4.000
	A_3	2.000	2.000	0.000	5.000
	P	-1.000	-3.000	-3.000	0.000
ϵ-multiplicative	A_1	1.000	1.000	0.900	4.000
	A_2	2.000	1.000	1.000	4.000
	A_3	2.000	1.500	1.000	6.000
	P	0.500	0.428	0.333	1.000
DoM	A_1	0.000	0.000	0.000	9.000
	A_2	2.000	0.000	0.000	9.000
	A_3	8.000	6.000	0.000	12.000
	P	0.000	0.000	0.000	0.000



Testing on more complex test problems

Multi-objective cases



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Testing on more complex test problems
Multi-objective cases

	MIP DoM correlation with		
Problem set	***-HV***	***IGD+***	***additive ϵ-indicator***
DTLZ1	0.896	0.347*	0.989
DTLZ2	0.770*	0.769*	0.783*
DTLZ3	0.964	0.887	0.998
DTLZ7	0.942	0.720*	0.502*
Combined DTLZ	**0.895**	**0.681**	**0.818**
WFG1	0.265*	0.019*	0.256*
WFG2	0.860	0.815*	0.626*
WFG3	0.945	0.747*	0.684*
WFG9	0.627*	0.093*	0.226*
Combined WFG	**0.674**	**0.372***	**0.448**
Combined All	**0.784**	**0.526**	**0.633**

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Testing on more complex test problems

Many-objective cases

Problem	*MIP-DoM(P, Q)*	*Value* L=100 M=5	L=170 M=10	L=240 M=15
DTLZ1	*(MOEA/D, NSGA-III)*	1463.461	560.182	594.894
	(NSGA-III, MOEA/D)	3401.332	6315.371	9944.266
DTLZ2	*(MOEA/D, NSGA-III)*	3.943	1.175	1.065
	(NSGA-III, MOEA/D)	3.804	8.972	16.665
DTLZ3	*(MOEA/D, NSGA-III)*	2839.942	1878.871	2040.851
	(NSGA-III, MOEA/D)	6418.077	15532.120	23735.510
DTLZ7	*(MOEA/D, NSGA-III)*	2.502	5.543	4.745
	(NSGA-III, MOEA/D)	1.148	2.948	2.950
WFG1	*(MOEA/D, NSGA-III)*	0.270	0.279	0.124
	(NSGA-III, MOEA/D)	0.464	0.707	0.775
WFG2	*(MOEA/D, NSGA-III)*	2.001	1.179	1.137
	(NSGA-III, MOEA/D)	1.571	2.423	3.665
WFG3	*(MOEA/D, NSGA-III)*	4.524	4.678	1.955
	(NSGA-III, MOEA/D)	5.170	14.160	22.981
WFG9	*(MOEA/D, NSGA-III)*	3.183	2.177	0.524
	(NSGA-III, MOEA/D)	4.450	13.583	31.296

Time-complexity estimates
Model size

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- Some examples:
  - Considering $M = 2$, $|\boldsymbol{P}| = |\boldsymbol{Q}| = 2$ the model has 16 continuous variables, 22 binary variables, and 60 constraints;
  - Considering $M = 5$, $|\boldsymbol{P}| = |\boldsymbol{Q}| = 200$, there are 202002 continuous variables, 440200 binary variables, and 923400 constraints.
- The Equations to calculate the number of continuous variables, binary variables, and constraints are detailed in the text.

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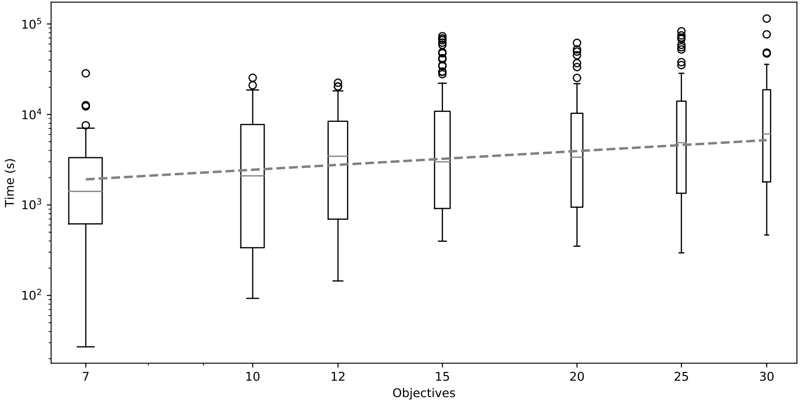
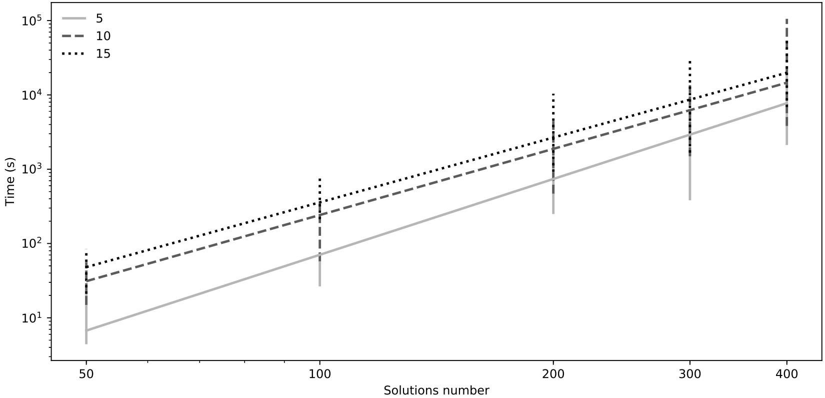
Time-complexity estimates
Experiment using DTLZ1 - Increasing the number of objectives



Linear time behavior $\approx O(M^{0.686})$.

Time-complexity estimates

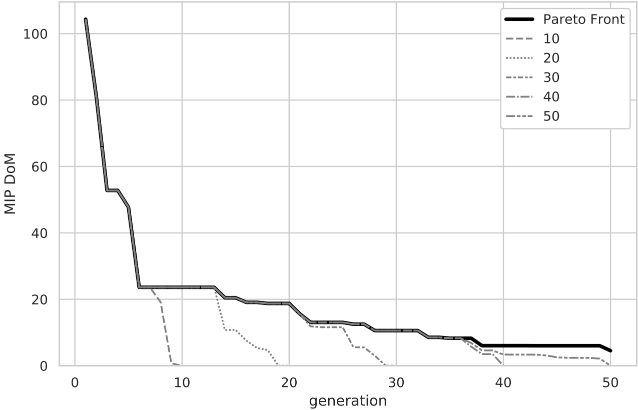
Experiment using DTLZ1 - Increasing the number of points



Polynomial time behavior with $\approx O(L^{3.311})$, $O(L^{2.888})$ and $O(L^{2.833})$.

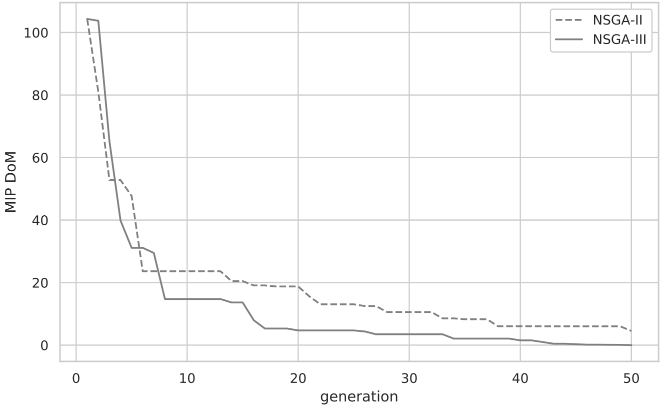
MIP-DoM as running performance indicator
Experiment using DTLZ1

Termination Criteria.



Monotonic decrease in each MIP-DoM curve.

MIP-DoM as running performance indicator
Experiment using DTLZ1 - Comparing two algorithms



MIP-DoM(NSGA-II, combined Pareto) in a dashed line;
MIP-DoM(NSGA-III, combined Pareto) in a solid line.

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Conclusion

- It does not require any reference point or any representative Pareto solution set;
- It is not affected by DRS;
- It overcomes the issue of information loss associated with the ϵ-indicator;
- Presumably, MIP-DoM is compatible in indicating four performance facets between two sets:
  - convergence,
  - spread,
  - uniformity, and
  - cardinality;
- It appears to have a highly Pearson correlation with HV;
- It presents a monotonic decrease in its value, when the first set (Q) is fixed and the second set (P) approaches towards the Pareto front;

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Conclusion

However, MIP-DoM quality indicator still present some drawbacks:

- MIP-DoM time calculation is more affected by the number of solutions in the set;
- As a running quality indicator, in some cases, the difference in MIP-DoM values between two consecutive generations are in decimal places;
- It demands an efficient MIP solver in its calculation.

Future research and final comments

Some possible directions

- An approximate MIP-DoM calculation using a cluster approach (the preliminary results are promissing);
- An accelerator operator based on the dominance move;
- The additional information generated by DoM can be applied such as ‘direction’ in the objective space search.

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Thank You!

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