Differential Evolution for Structural Multi-Objective Optimization

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Outline

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Introduction

- Differential Evolution (DE) is a popular evolutionary algorithm, proposed by Storn and Price (1995, 1997), aimed at solving continuous optimization problems.
- Basic DE stands out to be a very simple algorithm whose implementation requires only a few lines of code, and the canonical DE requires very few control parameters: the population size (NP), the crossover rate (CR), and the scale factor (F).
- DE exhibits remarkable performance while optimizing a wide variety of objective functions in terms of final accuracy, computational speed, and robustness.
- DE have been securing front ranks in various competitions among EAs organized under the IEEE Congress on Evolutionary Computation (CEC).

Introduction

- The total number of citations of DE since 1996 are recorded as 20366 till date as per Google Scholar citation and 7 of its variants has citations above 500. Bilal et al. (2020).

DE and its variants with citations above 500.

Variants	Year	Number of citation
Basic Differential Evolution (DE) (Storn and Price, 1997)	1996	20 366
Self-Adaptive Differential Evolution (SaDE) (Qin and Suganthan, 2005)	2005	2410
Adaptive Differential Evolution with Optional External Archive (JADE) (Zhang and Sanderson, 2009)	2009	1888
Opposition Based Differential Evolution (ODE) (Rahnamayan et al., 2008)	2008	1296
Neighborhood Based Differential Evolution (NDE) (Das et al., 2009)	2009	960
Composite Differential Evolution (CoDE) (Wang et al., 2011)	2011	898
Fuzzy Adaptive Differential Evolution (FADE) (Liu and Lampinen, 2005)	2005	857
Generalized Differential Evolution (GDE3) (Kukkonen and Lampinen, 2005)	2005	523

Figure 1.1: Extracted from Bilal et al. (2020)

Performance of DE variants in different CEC competition.

DE variant	CEC competition	Rank
SaDE (Qin and Suganthan, 2005)	CEC 2005	3rd
ε_DE (Takahama and Sakai, 2006)	CEC 2006	1st
GDE3 (Kukkonen and Lampinen, 2005)	CEC 2007	2nd
jDEdynNP-F (Brest et al., 2008)	CEC 2008	3rd
jDE (Brest et al., 2006)	CEC 2009	1st
εDEg (Takahama and Sakai, 2010)	CEC 2010	1st
DE-ACr (Reynoso-Meza et al., 2011)	CEC 2011	2nd
SHADE (Tanabe and Fukunaga, 2013)	CEC 2013	4th
L-SHADE (Tanabe and Fukunaga, 2014)	CEC 2014	1st
SPS-L-SHADE-EIG (Guo et al., 2015)	CEC 2015	1st
L-SHADE-Epsin (Awad et al., 2016a)	CEC 2016	1st
L-SHADE-cnEpsin (Awad et al., 2017a)	CEC 2017	3rd
L-SHADE-RSP (Akhmedova et al., 2018)	CEC 2018	2nd

Figure 1.2: Extracted from Bilal et al. (2020)

Introduction

- Structural engineering deals with the design and construction, which includes works such as bridges, roads, buildings, and trusses. By determining the stability of structures, it can materialize, among others, works for livability of the society against environmental phenomena.

- Structural design optimization problems are usually characterized by the presence of multiple conflicting objectives, as to get the minimum investment cost and the maximum safety of the final design.

Introduction

- Zavala et al. (2014) reviews the latest developments in the field of multi-objective metaheuristics for solving design problems focusing on the optimization of the topology, shape, and sizing of civil engineering structures. The paper ends by addressing a number of relevant and open issues that can be the subject of further research.
- “*Many of the problems reported in the specialized literature are encoded using real numbers for all the decision variables, and we found no study in which differential evolution was used to solve any of these problems. This is rather surprising if we consider that differential evolution is a very powerful approach for solving problems in which all the decision variables are real numbers. Some of the multi-objective metaheuristics based on differential evolution that could be used for these problems are GDE3 (Kukkonen and Lampinen 2005) and MOSADE (Huang et al. 2009).*” Zavala et al. (2014).

Introduction

- "*Another aspect that has been only scarcely explored in multi-objective structural optimization using metaheuristics, is the incorporation of user's preferences in the search engine. These preferences allow, for example, to focus the search into a specific region of the Pareto front, and also helps the decision maker to choose one (or very few) solution from the many that a multi-objective metaheuristic normally generates.*" Zavala et al. (2014).
- As suggested by Zavala et al. (2014), the Third Evolution Step of Generalized Differential Evolution (GDE3) proposed by Kukkonen and Lampinen (2005) was adopted to solve structural multi-objective optimization problems, with and without incorporation of user's preferences in the search engine. We also recently evaluated the performance of other DE-based MOEAs to solve structural multi-objective optimization problems.

Differential Evolution

- Differential Evolution (DE) utilizes NP D-dimensional parameter vectors $x_{i,G}, i = 1, \ldots, NP$ as a population for each generation G.

- For each target vector $x_{i,G}, i = 1, \ldots, NP$, a mutant vector is generated according to $v_{i,G+1} = x_{r1,G} + F(x_{r2,G} - x_{r3,G})$ with random indexes $r1, r2, r3 \in \{1, \ldots, NP\}$ mutually different and different from the running index i. $F > 0$ is a real factor which controls the amplification of the differential variation.

- The trial vector $u_{i,G+1} = (u_{1i,G+1}, \ldots, u_{1D,G+1})$ is formed, where

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1}, & \text{if } randb(j) \leq CR \text{ or } j = rnbr(i) \\ x_{ji,G}, & \text{if } randb(j) > CR \text{ and } j \neq rnbr(i) \end{cases}$$

Differential Evolution

- $0 \leq randb(j) \leq 1$ is the jth random number and $0 \leq CR \leq 1$ is the crossover rate. $rnbr(i) \in \{1, \ldots, NP\}$ is a randomly chosen index which ensures that $u_{i,G+1}$ gets at least one parameter from $v_{i,G+1}$.

- To decide whether or not it should become a member of generation G+1, the trial vector $u_{i,G+1}$ is compared to the target vector $x_{i,G}$. If $u_{i,G+1}$ yields a smaller cost function value than $x_{i,G}$, then $x_{i,G+1}$ is set to $u_{i,G+1}$; otherwise, the old value $x_{i,G}$ is retained.

GDE3

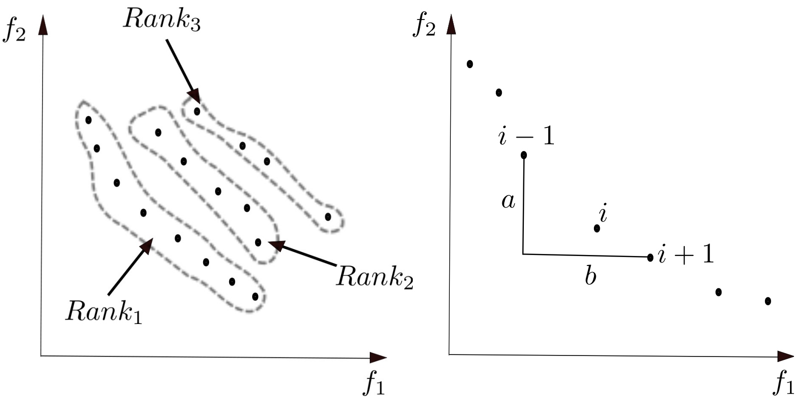
- GDE3 extended basic DE to constrained multi-objective optimization problems. Basically, the change consists of the selection criteria between $u_{i,G+1}$ and $x_{i,G}$. GDE3 uses the constraint domination concept.

- **x** constraint dominates **y** (denoted by $\mathbf{x} \succ_c \mathbf{y}$) if one, and only one, of the following conditions is true:
  - Both are infeasible and $\mathbf{x} \succ \mathbf{y}$ in the constraint function violation space.
  - **x** is feasible and **y** is infeasible.
  - **x** and **y** are feasible and $\mathbf{x} \succ \mathbf{y}$ in the objective function space.

- $u_{i,G+1}$ is selected if $u_{i,G+1} \succ_c x_{i,G}$. If $x_{i,G} \succ_c u_{i,G+1}$, $u_{i,G+1}$ is discarded and $x_{i,G}$ remains. Otherwise, both are included in the population.

GDE3

The size of the population is reduced using Non-dominated Ranking and Crowding Distance schemes when it is greater than NP.



(a) Non-dominated Ranking Scheme (b) The Crowding Distance Metric

GDE3

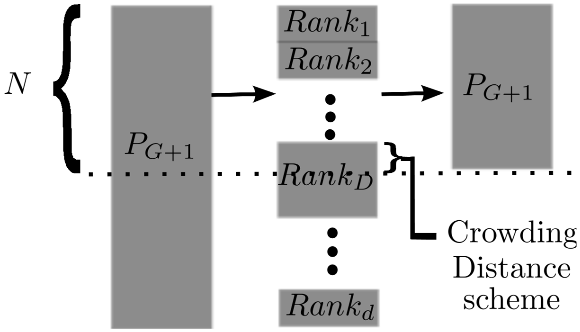


Figure 2.1: Reduction Size Procedure for P_{G+1} based on Non-dominated Ranking and Crowding Distance schemes.

Articles

- Vargas, D. E., Lemonge, A. C., Barbosa, H. J., and Bernardino, H. S. (2019). Differential evolution with the adaptive penalty method for structural multi-objective optimization. *Optimization and Engineering*, 20(1):65–88.

- Vargas, D. E., Lemonge, A. C., Barbosa, H. J., and Bernardino, H. S.(2021). Solving multi-objective structural optimization problems using GDE3 and NSGA-II with reference points. *Engineering Structures*, 239:112187.

Articles

- Lemonge, A. C., Carvalho, J. P., Hallak, P. H., and Vargas, D. (2021). Multi-objective truss structural optimization considering natural frequencies of vibration and global stability.*Expert Systems with Applications*, 165:113777.

- Carvalho, J. P. G., Érica C.R. Carvalho, Vargas, D. E., Hallak, P. H., Lima,B. S., and Lemonge, A. C. (2021). Multi-objective optimum design of truss structures using differential evolution algorithms. *Computers Structures*, 252:106544.

Vargas et al. (2019)

SpringerLink

Research Article | Published: 22 June 2018

Differential evolution with the adaptive penalty method for structural multi-objective optimization

Dênis E. C. Vargas, Afonso C. C. Lemonge, Helio J. C. Barbosa & Heder S. Bernardino

Optimization and Engineering **20**, 65–88 (2019) | Cite this article

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Multi-objective Structural Optimization Problems

- The multi-objective structural optimization problem solved here consists in finding a set of decision variables $\mathbf{x} = (x_1, \ldots, x_n)$, corresponding to the cross-sectional areas of the bars $(A_1, \ldots, A_n)$ of the truss, which minimize both the structure's weight and the maximum displacement of its nodes. The problem can be formulated as

$$\begin{array}{ll} \min & f_1(\mathbf{x}) = \sum_{j=1}^{n} \rho A_j L_j \quad \text{and} \quad f_2(\mathbf{x}) = \max(|u_{il}|) \\ \text{s.t.} & |s_{jl}| \leq s_{adm} \end{array}$$

$$j = 1, \ldots, n \quad i = 1, \ldots, M \quad l = 1, \ldots, N_L$$

Multi-objective Structural Optimization Problems

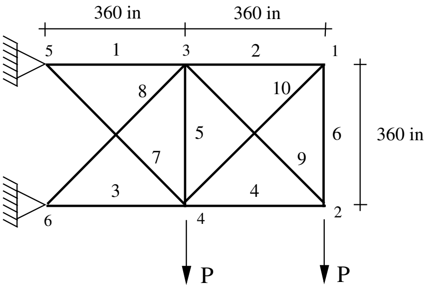
- $s_{jl}(\mathbf{x}) = E\varepsilon_{jl}(\mathbf{u}(\mathbf{x}))$ are the limits of the stress (constraints);
- E is the Young's modulus;
- n is the total number of bars in the truss structure;
- M is the number of degrees of freedom;
- N_L is the number of load cases applied to the structure;
- L_j is the length of the j-th bar;
- ρ is the density of the material;
- u_{il} is the nodal displacement of the i-th degree of freedom;
- s_{jl} is the stress of the j-th bar in the l-th load case;
- s_{adm} is the allowed stress.

Multi-objective Structural Optimization Problems

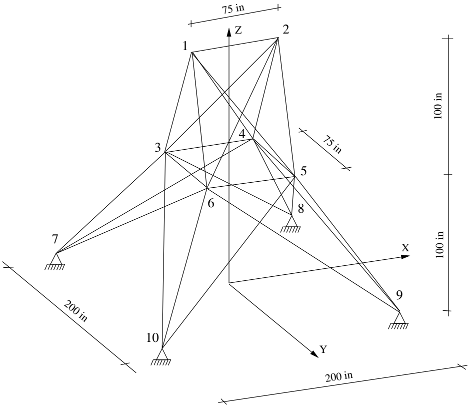
- Five well-known structural multi-objective optimization problems, corresponding to the 10-, 25-, 60-, 72-, and 942-bar trusses are tackled here.

- Both the discrete and the continuous cases (d/c) of all problems are considered.

Multi-objective Structural Optimization Problems

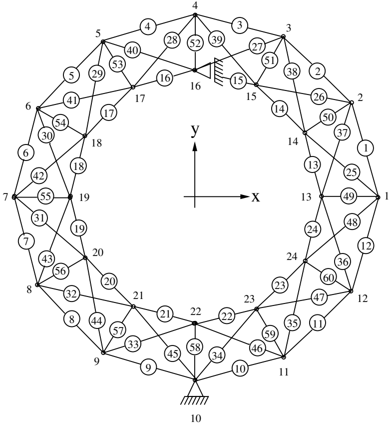


(a) 10-bar Truss.

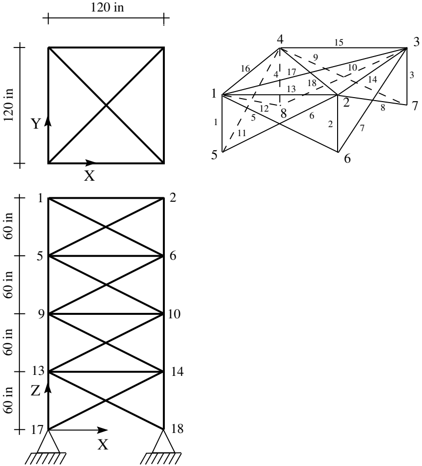


(b) 25-bar Truss.

Multi-objective Structural Optimization Problems

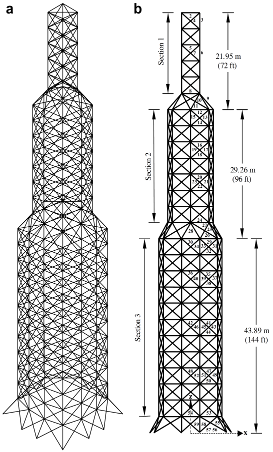


(a) 60-bar Truss.

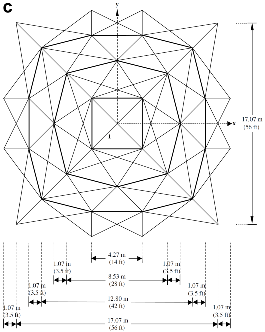


(b) 72-bar Truss.

Multi-objective Structural Optimization Problems



(a) 942-bar Truss - side view



(b) 942-bar Truss - top view

The Adaptive Penalty Method (APM)

- An alternative constraint handling technique was adopted: the Adaptive Penalty Method (APM) proposed by Barbosa and Lemonge (2002).
- Introducing a penalty function in the evaluation of the candidate solutions is a common way to handle constraints when using nature inspired techniques for optimization. However, the penalty coefficients are highly problem dependent and need to be tuned for each application.
- The APM aims at alleviating the user from the task of defining those values. The technique automatically sets those values using feedback from the search process. The idea is to observe how each constraint is being violated and set a higher penalty coefficient to those constraints which seem to be more difficult to satisfy.

The Adaptive Penalty Method (APM)

$f_{\text{fitness}}(\mathbf{x})$ of a given individual can be written as

$$f_{\text{fitness}}(\mathbf{x}) = \begin{cases} f(\mathbf{x}), & \text{if } \mathbf{x} \text{ is feasible,} \\ \overline{f}(\mathbf{x}) + \sum_{j=1}^{J} k_j v_j(\mathbf{x}) & \text{otherwise,} \end{cases}$$

where

$$\overline{f}(\mathbf{x}) = \begin{cases} f(\mathbf{x}), & \text{if } f(\mathbf{x}) > \langle f(\mathbf{x}) \rangle, \\ \langle f(\mathbf{x}) \rangle & \text{otherwise,} \end{cases}$$

and $\langle f(\mathbf{x}) \rangle$ is the average value of the objective function of the solutions in the current population.

The Adaptive Penalty Method (APM)

The penalty coefficient k_j, corresponding to the j-th constraint, is defined at every generation by

$$k_j = |\langle f(\mathbf{x}) \rangle| \frac{\langle v_j(\mathbf{x}) \rangle}{\sum_{l=1}^{J} [\langle v_l(\mathbf{x}) \rangle]^2},$$

where $v_j(\mathbf{x})$ is the violation of the j-th constraint averaged over the current population.

Performance Profiles

Performance profiles introduced by Dolan and More (2002): an analytical tool that makes it easier to visualize and to interpret the results of experiments. Let P be a set of n_p problems, S a set of algorithms, and $t_{p,s}$ any metric evaluated in problem p by algorithm s. The performance ratio is defined as

$$r_{p,s} = \frac{t_{p,s}}{min\{t_{p,s}, s \in S\}}.$$

Given the definition of $r_{p,s}$, the performance profile $\rho_s(\tau)$ is defined as the probability that the performance ratio $r_{p,s}$ of algorithm $s \in S$ is within a factor $\tau \geq 1$ of the best possible ratio. That is,

$$\rho_s(\tau) = \frac{1}{n_p}|\{p \in P : r_{p,s} \leq \tau\}|.$$

Performance Profiles

- So, for any given algorithm s, the performance profiles plot for each value of a positive factor τ is the percentage of problems from a given problem set on which the performance of s is within a factor of τ of the best performance of any algorithm on this problem.

- Also, Barbosa et al. (2010) indicated that the area under the performance profiles curves is an overall performance indicator for a algorithm in a problem set.

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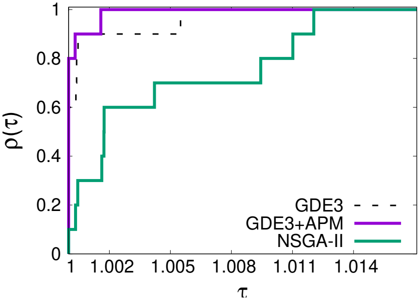
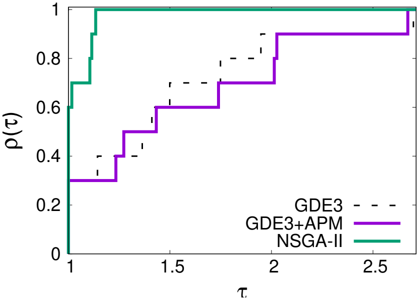
Results and Discussion



(a) HV: GDE3+APM=1; GDE3=0.9594; NSGA-II=0.6567



(b) IGD: GDE3+APM=0.6980; GDE3=0.7326; NSGA-II=1

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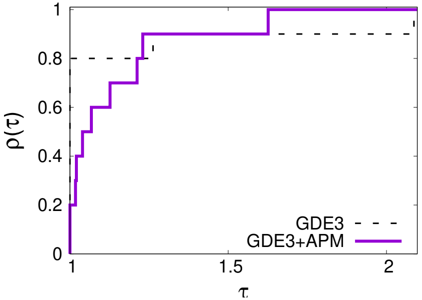
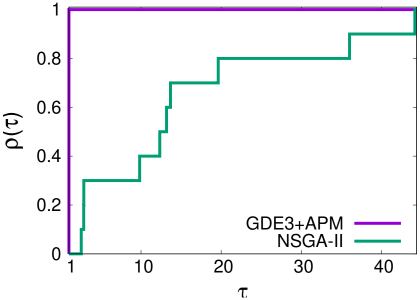
Results and Discussion

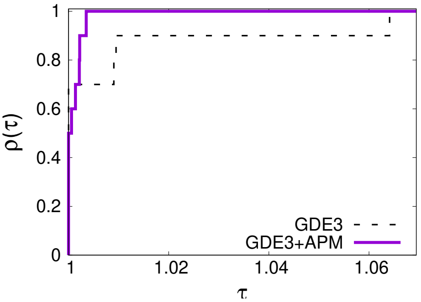


(a) CM: GDE3+APM=1; GDE3=0.9990

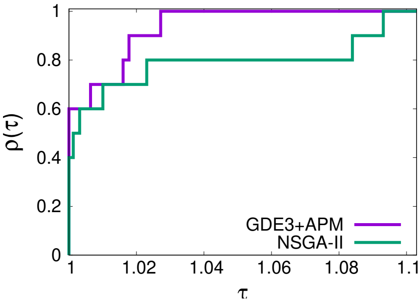


(b) CM: GDE3+APM=1; NSGA-II=0.6593

Results and Discussion

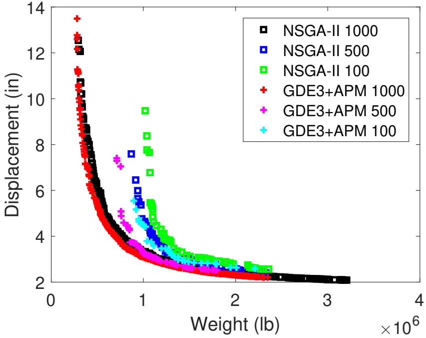


(a) I_ϵ: GDE3+APM=1; GDE3=0.8847

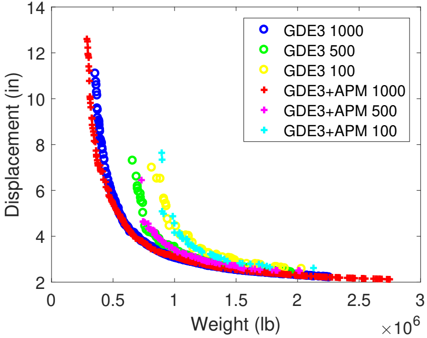


(b) I_ϵ: GDE3+APM=1; NSGA-II=0.8197

Results and Discussion



(a) Discrete Case



(b) Continuous Case

Figure 3.1: The obtained Pareto-optimal front with 1000 (Final Stage), 500 (Middle Stage) and 100 (Early Stage) generations of the GDE3+APM, GDE3 and NSGA-II algorithms in a given independent run for the 942-bar truss problem.

Results and Discussion

- It was observed that GDE3 and GDE3+APM perform better than NSGA-II for all problems analyzed here in all metrics adopted, except with respect to IGD.
- The APM allowed GDE3 to obtain solutions in extreme Pareto front, specially in the case of the 942-bar truss (the largest problem tackled here).
- GDE3+APM is promising when solving structural multi-objective optimization problems, showing competitive results.

Vargas et al. (2021)

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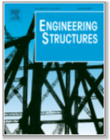


Solving multi-objective structural optimization problems using GDE3 and NSGA-II with reference points

Dênis E.C. Vargas [a], Afonso C.C. Lemonge [b], Helio J.C. Barbosa [c, d], Heder S. Bernardino [c]

DM's Preferences and Reference Points

- Most of the MOEAs focus on attaining all Pareto optimal solutions possible, and then the decision-maker (DM) choose a single solution that satisfies his/her preferences (usually, only one of Pareto optimal solutions is chosen by the DM).
- Procedures which incorporate the DM's preferences into such algorithms to drive the search for the Pareto optimal region based on the DM's desires have attracted the interest of researchers. The objective is to use that information to drive the search to a more relevant area.

DM's Preferences and Reference Points

- Purshouse et al. (2014) reviewed techniques which have combined MOEAs and multiple criteria decision making, and defined three classes of these hybrid techniques:
  - *a posteriori*, when the DM's preferences are incorporated after the search;
  - interactive, when the DM's preferences are incorporated progressively during the optimization process;
  - *a priori*, when the DM's preferences are incorporated before starting the search process.

DM's Preferences and Reference Points

- Among the most popular *a priori* techniques are those based on the reference point, i.e., those that search for solutions close to the DM specified aspiration levels for each objective, of which the reference point based NSGA-II (R-NSGA-II) proposed by Deb and Sundar (2006) is very well-known.

- New Crowding Distance operator in R-NSGA-II: the weighted normalized Euclidean distance for each reference point $\overline{z} = (z_1, ..., z_m)$ of each solution **x** of the front, mathematically defined as:

$$d_{\mathbf{x},\overline{z}} = \sqrt{\sum_{k=1}^{m} w_k \left(\frac{f_k(\mathbf{x}) - \overline{z_k}}{f_k^{MAX} - f_k^{MIN}} \right)^2}$$

DM's Preferences and Reference Points

A user-defined parameter ε controls the diversity.

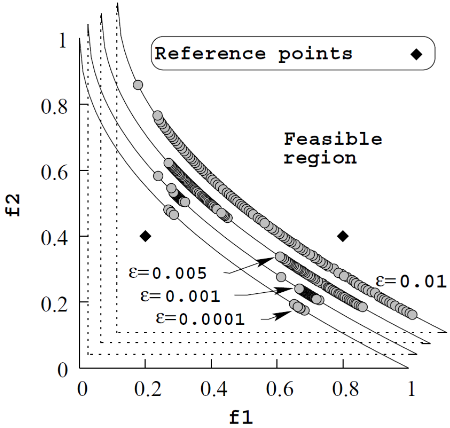


Figure 3.2: Effect of ε in obtaining varying spread of preferred solutions on ZDT1 (Deb and Sundar, 2006)

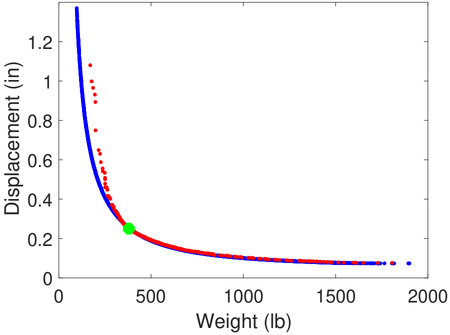
DM's Preferences and Reference Points

- The use of an *a priori* reference point (aspiration levels for each objective) fits well to situations where the DM has already accumulated some experience, as in the Multi-objective Structural Optimization Problems (MOSOPs) considered here.
- We propose here to use *a priori* reference points as DM's preferences information with all MOEAs and MOSOPs from Vargas et al. (2019).
- The reference points are defined before the algorithm starts, and they are kept fixed during the search process.
- We also propose here R-GDE3+APM and R-GDE3, respectively, GDE3+APM and GDE3 algorithms using preference information in the same way as R-NSGA-II.

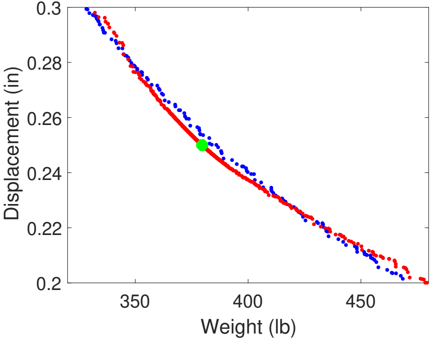
DM's Preferences and Reference Points

- The reference point for each problem was adopted as the best solutions shown in Silva et al. (2013) for the related structural single-objective optimization problems.
- The DM's Region of Interest (ROI) adopted for each problem was the region of the reference point neighborhood where the MOEAs with Reference Point dominates the Pareto front from Vargas et al. (2019).

DM's Preferences and Reference Points



(a) 72-bar truss continuous case

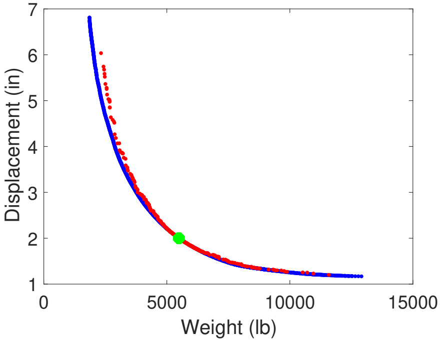


(b) DM's ROI

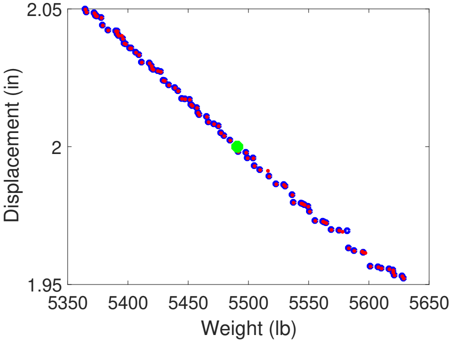
DM’s Preferences and Reference Points

- R-GDE3+APM, R-GDE3, and R-NSGA-II were applied to the continuous concerning the 10-, and 25-bar trusses, and to both discrete and continuous MOSOPs of the 60-, 72-, and 942-bar trusses.
- Although Vargas et al. (2019) also analyzed the discrete cases of the 10-bar and 25-bar trusses, these problems were excluded from this paper as R-GDE3+APM, R-GDE3, and R-NSGA-II do not find any solution that dominates those obtained by GDE3+APM, GDE3, and NSGA-II in the neighborhood of the respective reference points.

DM's Preferences and Reference Points

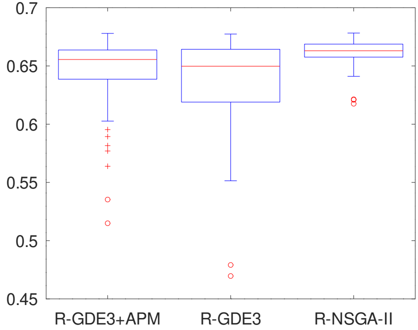


(a) 10-bar truss discrete case

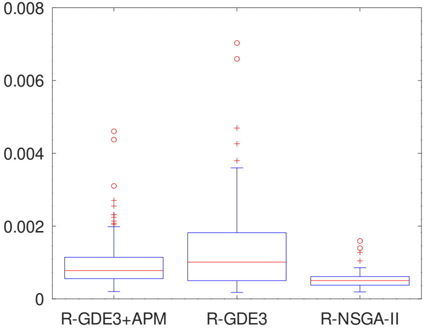


(b) Any solution obtained with Reference Point information dominated the complete Pareto Front

Results and Discussion

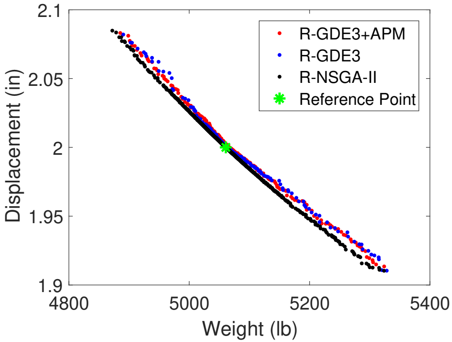


(a) 60-bar truss discrete case (HV)

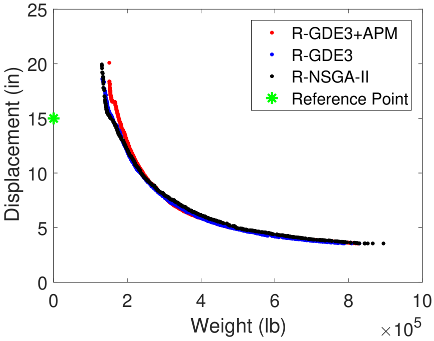


(b) 60-bar truss discrete case (IGD)

Results and Discussion



(a) 10-bar truss continuous case



(b) 942-bar truss continuous case

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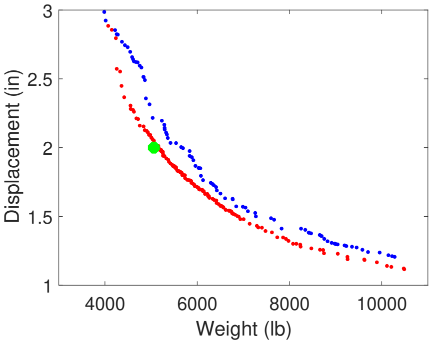
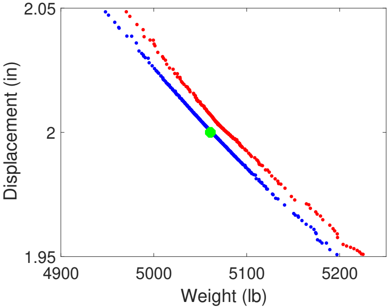
Results and Discussion



(a) 50^{th} generation



(b) 250^{th} generation

Figure 3.3: R-NSGA-II (blue points) and R-GDE3+APM (red points) in the 10-bar truss problem continuous case.

Results and Discussion

- Wilcoxon rank-sum test was applied.
- R-GDE3+APM performed better than R-GDE3, but without differences statistically significant in most cases.
- R-NSGA-II obtained the best results with differences statistically significant in most cases.
- One of the reasons is the SBX, which tends to generate offspring close to the parents, causing the ability of R-NSGA-II to find Pareto optimal solutions much closer to the reference point than those obtained by R-GDE3+APM and R-GDE3.

Lemonge et al. (2021)

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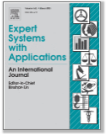

Multi-objective truss structural optimization considering natural frequencies of vibration and global stability

Afonso C.C. Lemonge [a], José P.G. Carvalho [b], Patrícia H. Hallak [a], Dênis.E.C. Vargas [c]

Multi-objective Structural Optimization Problems

- The first multi-objective structural optimization (MOSO1) is written as:

$$\begin{aligned} \min \quad W(\mathbf{x}) \quad \text{and} \quad \max \quad f_1(\mathbf{x}) \\ \text{s.t.} \qquad\qquad \sigma_i(\mathbf{x}) \leq \overline{\sigma} \\ u_j(\mathbf{x}) \leq \overline{u} \\ \lambda_1(\mathbf{x}) \geq \overline{\lambda} \\ \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{aligned}$$

where $f_1(\mathbf{x})$ is the first natural frequency of vibration, $\sigma_i(\mathbf{x})$ is the axial stress at the i-th bar, $u_j(\mathbf{x})$ is the displacement at the j-th node and $\lambda_1(\mathbf{x})$ is the smallest load factor with respect to the maximum elastic critical load able to be applied to the structure. The search space of the design variables is defined by the lower $\mathbf{x}^L$ and upper $\mathbf{x}^U$ bounds, respectively.

Multi-objective Structural Optimization Problems

- The second multi-objective structural optimization (MOSO2) is written as:

$$\begin{array}{cc} \min \quad W(\mathbf{x}) \quad \text{and} \quad \max \quad \lambda_1(\mathbf{x}) \\ \text{s.t.} \qquad\qquad \sigma_i(\mathbf{x}) \le \overline{\sigma} \\ u_j(\mathbf{x}) \le \overline{u} \\ f_1(\mathbf{x}) \ge \overline{f} \\ \mathbf{x}^L \le \mathbf{x} \le \mathbf{x}^U \end{array}$$

where $\lambda_1(\mathbf{x})$ is the smallest load factor with respect to the maximum elastic critical load able to be applied to the structure, $u_j(\mathbf{x})$ is the displacement at the j-th node and $f_1(\mathbf{x})$ is the first natural frequency of vibration. The search space of the design variables is defined by the lower $\mathbf{x}^L$ and upper $\mathbf{x}^U$ bounds, respectively.

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Multi-objective Structural Optimization Problems

- The third multi-objective structural optimization (MOSO3) is written as:

$$\begin{aligned} \min \quad W(\mathbf{x}) \quad \text{and} \quad \min \quad u_{\mathsf{max}}(\mathbf{x}) \\ \text{s.t.} \qquad \sigma_i(\mathbf{x}) \leq \overline{\sigma} \\ f_1(\mathbf{x}) \geq \overline{f} \\ \lambda_1(\mathbf{x}) \geq \overline{\lambda} \\ \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{aligned}$$

where $u_{\mathsf{max}}(\mathbf{x})$ is the maximum nodal displacement of the structure, $\sigma_i(\mathbf{x})$ is the axial stress at the i-th bar and $\lambda_1(\mathbf{x})$ is the smallest load factor with respect to maximum elastic critical load able to be applied to the structure. The search space of the design variables is defined by the lower $\mathbf{x}^L$ and upper $\mathbf{x}^U$ bounds, respectively.

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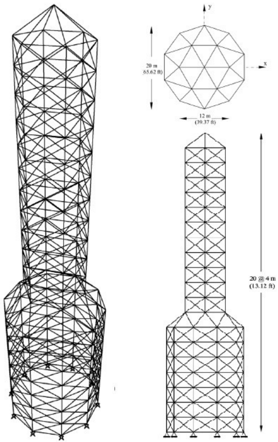
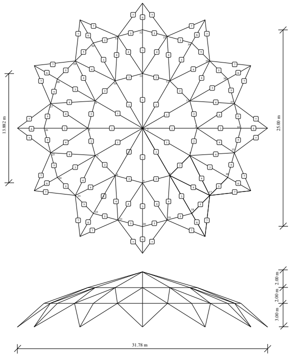
Multi-objective Structural Optimization Problems

- Four well-known structural multi-objective optimization problems, corresponding to the 10-, 72-, and 582-bar trusses are tackled here, besides the 120-bar truss dome.

Multi-objective Structural Optimization Problems



(a) 582-bar truss

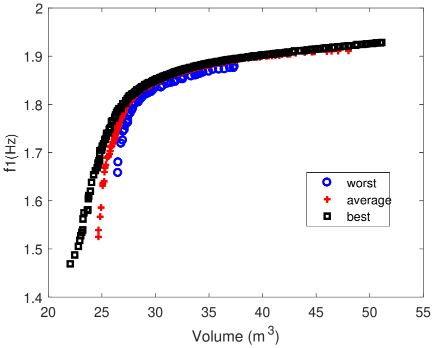


(b) 120-bar truss dome

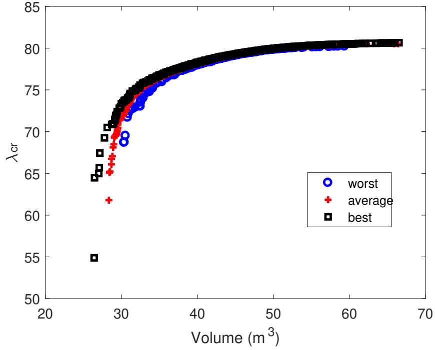
Algorithm, Performance Metric, and Multicriteria Decision Method (MCDM)

- GDE3+APM and Empirical Attainment Function (EAF) was adopted.
- In this paper, a Multicriteria Decision Method (MCDM) using a Multicriteria Tournament Decision (MTD) is adopted in order to illustrate different scenarios defined by the DM in order to explore solutions at the Pareto frontier.
- The MTD is a tournament-based method that ranks the best and the worst possible solutions in the Pareto frontier according to their objectives and preferences (weights) established by the decision maker.

Results and Discussion

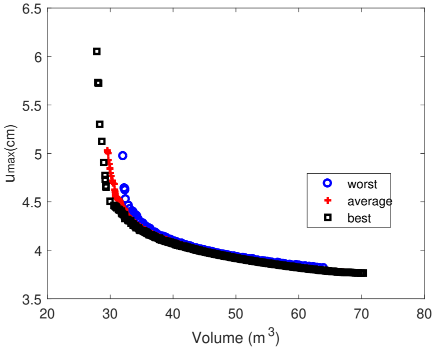


(a) MOSO1 - best=1, average=0.95240 and worst=0.86715



(b) MOSO2 - best=1, average=0.96388 and worst=0.91938

Figure 3.4: EAFs and the respective relative best, average and worst hypervolumes for the 582-bar truss.



(a) MOSO3 - best=1, average=0.97536 and worst=0.92119

Figure 3.5: EAFs and the respective relative best, average and worst hypervolumes for the 582-bar truss.

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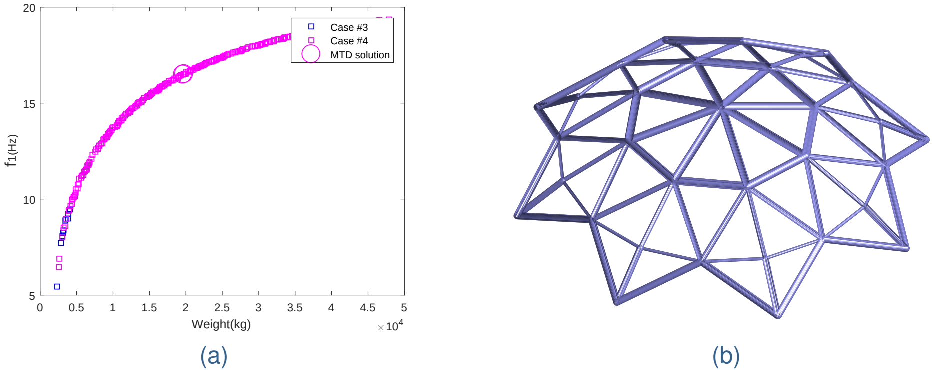

Figure 3.6: 120-bar truss dome: Pareto and the extracted solution setting w_1 = 0.5 and w_2 = 0.5. The preferred solution defined by the DM (MTD solution) is marked with an open circle.

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Results and Discussion

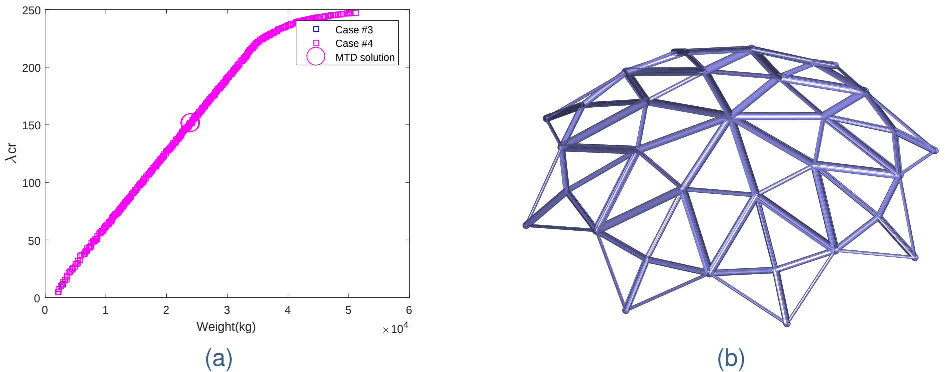
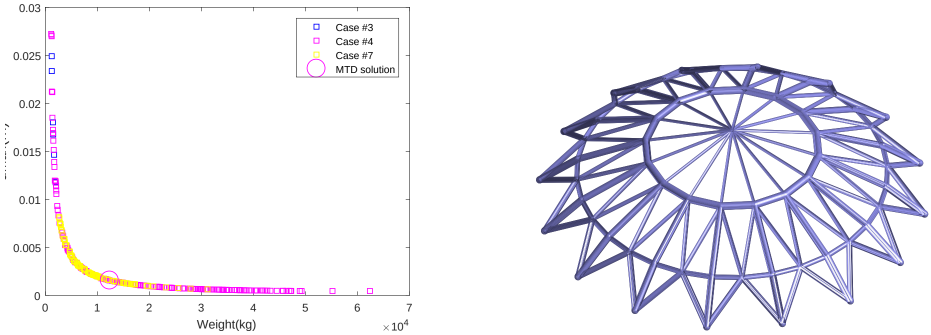


Figure 3.7: 120-bar truss dome: Pareto and the extracted solution setting $w_1 = 0.5$ and $w_2 = 0.5$. The preferred solution defined by the DM (MTD solution) is marked with an open circle.

Results and Discussion



(a) (b)

Figure 3.8: 120-bar truss dome: Pareto and the extracted solution setting w_1 = 0.5 and w_2 = 0.5. The preferred solution defined by the DM (MTD solution) is marked with an open circle.

Results and Discussion

- GDE3+APM presented very good performance in solving the optimization problems presented in this paper, especially when observing the EAFs curves referring to the best, average and worst Paretos where the hypervolumes, in these three metrics, were very similar.

- The great majority of the problems analyzed in this paper are new proposals of multi-objective structural optimization problems. It can be justified since for the traditional examples found in the literature such as 10-, 72-, 582- or 120-bar trusses the formulations do not consider stress, nodal displacements, natural frequencies of vibration, or the global stability as constraints or objective functions.

Carvalho et al. (2021)

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Volume 252, August 2021, 106544

Multi-objective optimum design of truss structures using differential evolution algorithms

José Pedro G. Carvalho [a], Érica C.R. Carvalho [b], Dênis E.C. Vargas [c], Patrícia H. Hallak [d], Beatriz S.L.P. Lima [a], Afonso C.C. Lemonge [d]

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Multi-objective Structural Optimization Problems

- The first problem (MOSOP1) is written as:

$$\begin{array}{rl} \min \quad W(\mathbf{x}) \quad , \quad \max \quad f_1(\mathbf{x}) \quad , & \min \quad u_{\text{max}}(\mathbf{x}) \\ \text{s.t.} & \sigma_i(\mathbf{x}) \leq \bar{\sigma} \\ & \lambda_1(\mathbf{x}) \geq \bar{\lambda} \\ & \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U, \end{array}$$

- The second problem (MOSOP2) is written as:

$$\begin{array}{rl} \min \quad W(\mathbf{x}) \quad , \quad \max \quad \lambda_1(\mathbf{x}) \quad , & \min \quad u_{\text{max}}(\mathbf{x}) \\ \text{s.t.} & \sigma_i(\mathbf{x}) \leq \bar{\sigma} \\ & f_1(\mathbf{x}) \geq \bar{\lambda} \\ & \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U, \end{array}$$

Multi-objective Structural Optimization Problems

- The third problem (MOSOP3) is written as:

$$\begin{aligned} \min \quad W(\mathbf{x}) \quad , \quad \max \quad f_1(\mathbf{x}) \quad , \quad \max \quad \lambda_1(\mathbf{x}) \\ \text{s.t.} \qquad \sigma_i(\mathbf{x}) \leq \overline{\sigma} \\ u_j(\mathbf{x}) \leq \overline{u} \\ \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U , \end{aligned}$$

- The fourth problem (MOSOP4) is written as:

$$\begin{aligned} \min \quad W(\mathbf{x}) \quad , \quad \max \quad f_1(\mathbf{x}) \quad , \quad \min \quad u_{max}(\mathbf{x}) \quad , \quad \max \quad \lambda_1(\mathbf{x}) \\ \text{s.t.} \qquad \sigma_i(\mathbf{x}) \leq \overline{\sigma} \\ \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U , \end{aligned}$$

- Seven MOSOPs, the 10-, 25-, 56-, 72-, 120-, and 582-bar trusses and a 33-bar ground-structure system, are analyzed.

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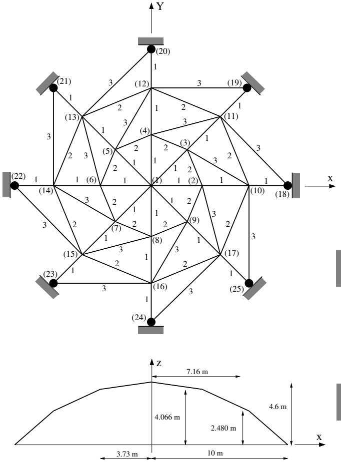
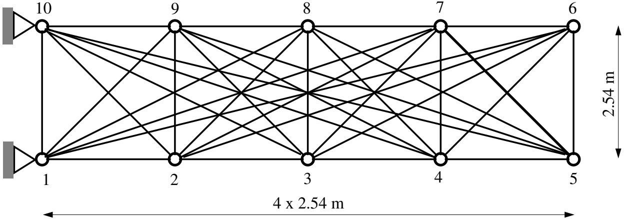
Multi-objective Structural Optimization Problems



(a) 56-bar truss



(b) 33-bar ground-structure system

Algorithms, Performance Metrics, and Multicriteria Decision Method (MCDM)

- The DE algorithms used to solve the MOSOPs formulated in this paper are GDE3+APM, the success history–based adaptive multi-objective differential evolution (SHAMODE) and SHAMODE with whale optimisation (SHAMODE-WO) (Panagant et al., 2019), and the multiobjective meta-heuristic with iterative parameter distribution estimation (MM-IPDE) (Wansasueb et al., 2020).
- To evaluate the performance of the algorithms, the spacing (S), the hypervolume (HV) and the empirical attainment function (EAF) are adopted.
- Once again, MCDM using a MTD is adopted to simulate DM's choices in order to explore solutions at the Pareto frontier.

Results and Discussion

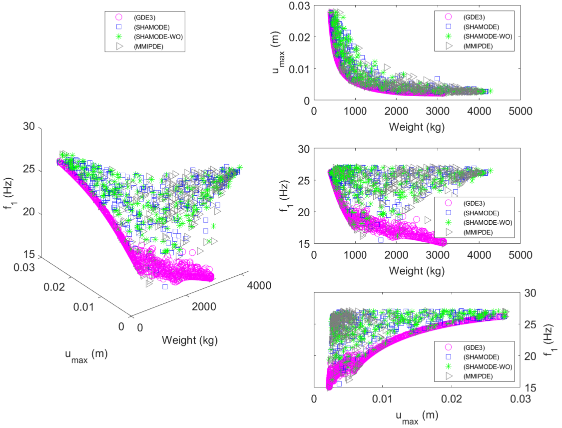


Figure 3.9: Results of the 56-bar truss: MOSOP1.

Results and Discussion

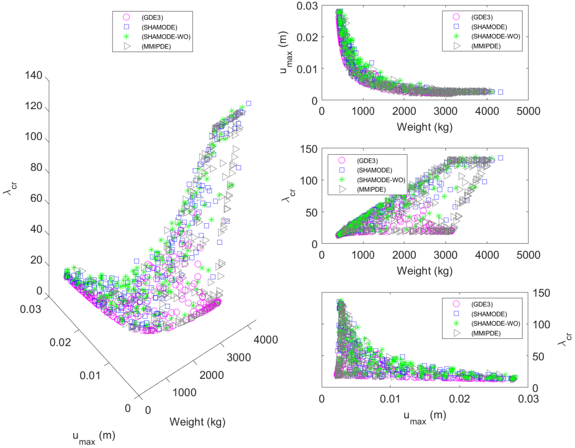


Figure 3.10: Results of the 56-bar truss: MOSOP2.

Results and Discussion

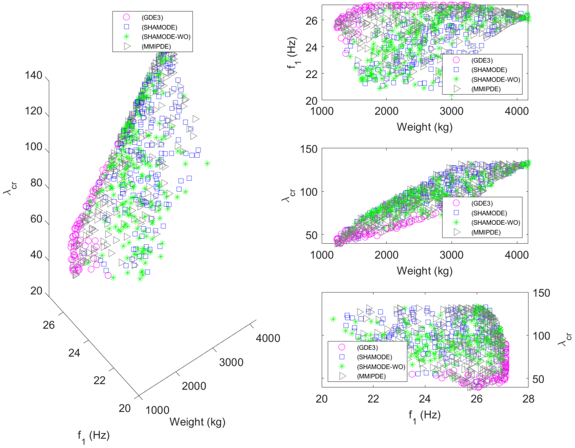


Figure 3.11: Results of the 56-bar truss: MOSOP3.

Results and Discussion

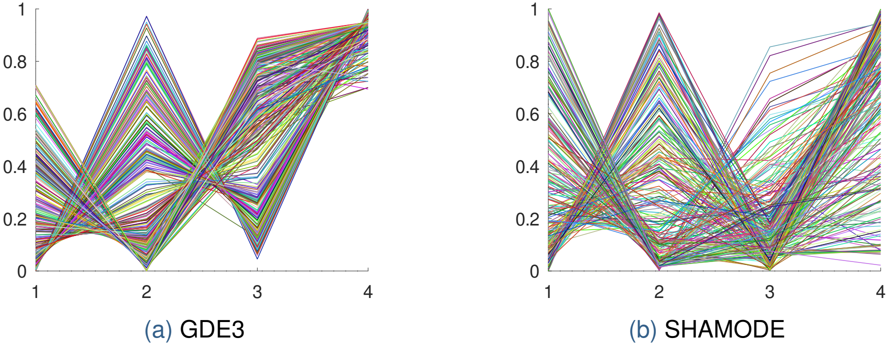
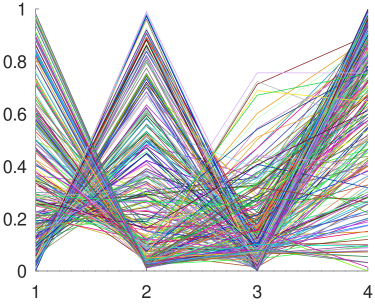
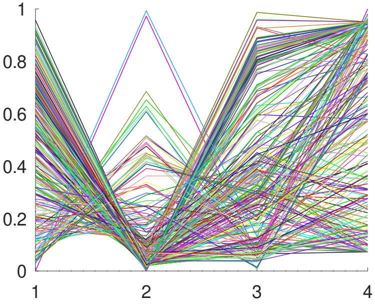


Figure 3.12: Parallel coordinates of Pareto front solutions (normalized to 0-1 range) on the 4-objective MOSOP4 for the 56-bar truss problem.

Results and Discussion



(a) SHAMODE-WO



(b) MM-IPDE

Figure 3.13: Parallel coordinates of Pareto front solutions (normalized to 0-1 range) on the 4-objective MOSOP4 for the 56-bar truss problem.

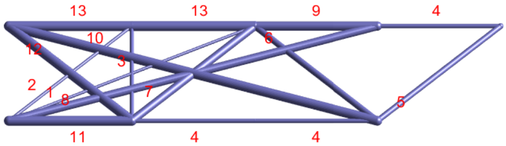


Figure 3.14: Extracted solution for the 33-bar ground-structure system (MOSOP1) setting w_1 = 0.6 (W), w_2 = 0.2 ($u_{\max}$), and w_3 = 0.2 (f_1) using MTD to simulate DM's choice.

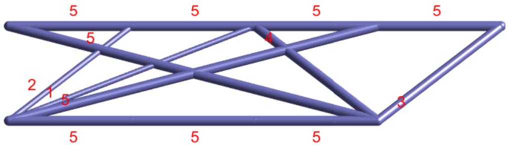


Figure 3.15: Extracted solution for the 33-bar ground-structure system (MOSOP2) setting $w_1 = 0.6$ (W), $w_2 = 0.2$ (λ_{crit}), and $w_3 = 0.2$ (u_{max}) using MTD to simulate DM's choice.

Results and Discussion

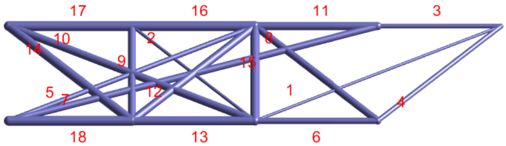
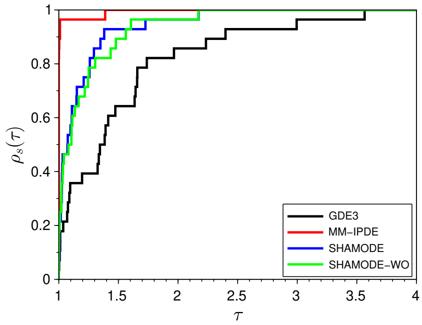
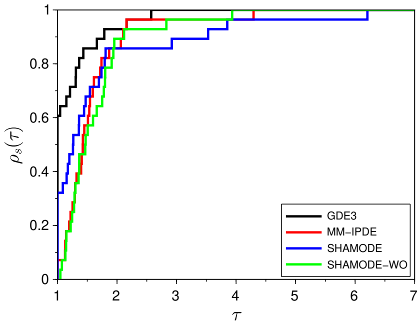


Figure 3.16: Extracted solution for the 33-bar ground-structure system (MOSOP3) setting w_1 = 0.6 (W), w_2 = 0.2 (f_1), and w_3 = 0.2 (λ_{crit}) using MTD to simulate DM's choice.

Results and Discussion



(a) $t_{p,s}(HVEAF_{\text{best}})$



(b) $t_{p,s}(SEAF_{\text{best}})$

Results and Discussion

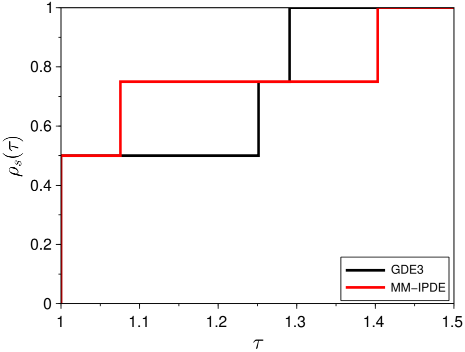


Figure 3.17: Performance profile curves of the overall metric $t_{p,s}(PP)$ between MM-IPDE and GDE3. The normalized areas under their curves are MM-IPDE (1) and GDE3 (0.94380), indicating that MM-IPDE has a good overall performance.

Results and Discussion

- This paper presented structural optimization problems with 3 and 4 conflicting objectives.

- Performance Profiles were adopted, indicating the MM-IPDE was the best one. According to the performance indicators used, it is important to note that the GDE3, SHAMODE, and SHAMODE-WO algorithms also proved to be competitive.

Future Research

- Evaluate these algorithms on other MOSOPs, such as the problems analyzed here with the addition of cardinality constraints (optimal grouping of bars).
- MOEAs combined with the DM's preferences inserted interactively (i.e., incorporating progressively during the search process).
- MOSOPs for other types of structures, such as spatial frames, large-scale ground-structures systems, and problems with a higher degree of complexity.
- Many-objective Structural Optimization Problems.
- New MOEAs.

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Thank You!

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